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# Forecasting Money Flow in East Java Using The Generalized Space-Time Autoregressive With Exogenous Variable Method

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**Abstract:**
The forecast is more accurate when involving an exogen, for example, the generalized spatio temporal autoregressive (GSTAR) model with exogenous variables (GSTARX). This study aims to obtain appropriate statistical values to identify autoregressive orders for time dependencies and order effects with exogenous variables such as Eid al-Fitr and the consumer price index (CPI) in the GSTARX model. Additionally, this study seeks to obtain a GSTARX model suitable for forecasting. The case study is the inflows and outflows in four locations in East Java. The simulation results show that the statistical values for identifying VAR order for time dependencies use the matrix partial cross-correlation function (MPCCF). Meanwhile, effect orders from exogenous variables in the two-level GSTARX model show CCF at level one. In addition, GLS (general least squares) produces a more efficient estimator than the OLS (ordinary least squares). The results of the case study show that the two-level GSTAR model forming inflow and outflow data are GSTARX (31) and GSTARX ([1,3]1) respectively. In addition, the comparison of forecast accuracy shows that, in general, the smallest root mean square error (RMSE) value in the two-level GSTARX model is the inverse distance weight.
**Keywords**: *GSTARX, Inflows, Outflows, Eid Al-Fitr, Consumer Price Index*

**Abstrak:**
Perkembangan pemodelan time series, terutama pemodelan untuk data multivariat, cukup pesat dan hasil akurasi ramalan lebih baik jika melibatkan suatu eksogen, salah satu contohnya adalah model Generalized Space-Time Autoregressive (GSTAR) dengan variabel eksogen (GSTARX). Pada penelitian ini ingin mendapatkan besaran statistik yang sesuai untuk mengidentifikasi order Autoregressive untuk dependensi waktu dan order efek dengan variabel eksogen berupa Hari Raya Idul Fitri dan Indeks Harga Konsumen pada model GSTARX. Selain itu, juga ingin mendapatkan model GSTARX yang sesuai untuk peramalan, serta membandingkan hasil akurasi dari peramalan model GSTARX dengan berbagai macam bobot lokasi pada peramalan pada studi kasus. Sebagai studi kasus adalah inflow dan outflow di empat Kantor Perwakilan Bank Indonesia Wilayah Jawa Timur. Hasil kajian simulasi menunjukkan bahwa esaran statistik untuk mengidentifikasi order VAR untuk dependensi waktu menggunakan MPCCF, sedangkan orde efek dari variabel eksogen pada model GSTARX dua level dengan CCF di Level 1. Sebagai tambahan, GSTARX-GLS menghasilkan estimator yang lebih efisien dibandingkan dengan model GSTARX-OLS. Hasil kajian terapan menunjukkan model GSTAR pada level dua yang terbentuk berturut-turut data inflow dan data outflow adalah GSTARX (31) dan GSTARX ([1,3]1) di empat lokasi tersebut. Sebagai tambahan, hasil perbandingan akurasi ramalan menunjukkan bahwa secara umum nilai RMSE terkecil pada model GSTARX dengan dua level terdapat pada bobot invers jarak.
**Kata Kunci:** *GSTARX; Inflow; Outflow; Hari Raya Idul Fitri; Indeks Harga Konsumen*

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## INTRODUCTION

Forecasting is an important activity in many economic sectors. Therefore, forecasting studies have been developed and applied to answer various needs[1] and assist in planning and informed decision-making. The autoregressive integrated moving average (ARIMA) is the most popular forecasting model. The enhanced ARIMA model involving exogenous variables is called ARIMAX, with X denoting an exogenous variable. In the metric form, X is known as the transfer function model[2]. In the non-metric form, X is known as the intervention model[3] and the calendar variation model[4,5,6].

Time series modeling, especially for multivariate data, is developing rapidly, for example, the generalized spatio temporal autoregressive (GSTAR) model[7]. Forecasts made by this model will be more accurate when involving an exogenous variable. However, the development of the GSTAR model involving an exogenous variable remains scarce. Also, the estimation of spatiotemporal modeling parameters is still limited by using ordinary least squares (OLS)[8]. Terzi believes that parameter estimation with OLS in the GSTAR model is inefficient because the residuals are correlated with each other. The alternative parameter estimation method for correlated residuals is generalized least squares (GLS).[9,10,11]

This study uses the GSTAR model with exogenous variables (GSTARX), especially with the GLS parameter estimation method (GSTARX-GLS henceforth). The aim is to obtain appropriate statistical values to identify autoregressive (AR) orders for time dependencies and effect orders with exogenous variables such as the Eid al-Fitr season and the consumer price index (CPI). The second aim is to discover the efficiency of the parameter estimator generated from GLS compared to OLS. In the case study, we tested the GSTARX model to reveal the forecasting efficiency and compared the accuracy results of the GSTARX model in various location weights.

The case study involves inflows and outflows data in East Java Province. The term inflows refer to the activities of commercial banks to deposit money to Bank Indonesia, while

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outflows mean the activities of commercial banks to withdraw money eligible for circulation from Bank Indonesia[12]. Inflow and outflow activities are carried out to maintain money availability in the community through all banks in Indonesia. The inflows and outflows in East Java increase significantly during festive seasons such as Eid al-Fitr[13]. This annual season gives a calendar variation effect, which is used as a non-metric exogen in this study. The increase in consumption during Eid al-Fitr festive affects the Consumer Price Index (CPI), which affects the amounts of inflows and outflows.

Research on inflows and outflows has been carried out widely with various methods[14,15,16] in various places[17,18,19]. However, forecasting using inflows and outflows data and the GSTARX method in East Java Province has not been conducted. East Java is one of the biggest economic centers in the eastern part of Indonesia. The province's population is mostly Muslim. Every year, Muslims in East Java celebrate Eid al-Fitr. The flows of money before and after the holiday season are massive, so it is necessary to make a forecast to improve the effectiveness and efficiency of money circulation.

The research limits the spatial order to the first order only because the areas under study are assumed to be in the same or adjacent area.

**METHODS**

Two research objectives in the research are a simulation to validate the proposed model and a case study to predict the inflows and outflows in four locations in East Java, Indonesia.

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## 1. Simulation Study

The simulation study is designed to generate six residual data $\boldsymbol{n_t}$, as shown in Figure 1. The generation data consists of three locations and $n$ = 300. Then $\boldsymbol{Y_t}$ is obtained by inputting the impulse response weights from $\boldsymbol{X_t}$, with the effect of all locations being uniform, using two scenarios as shown in Figure 2. Scenarios 1 and 2 can be seen in Equations (1) and (2).

$$\boldsymbol{y}_t = (\boldsymbol{\omega_0} - \boldsymbol{\omega_1} B)\boldsymbol{x}_{t-1} + \boldsymbol{n}_t \quad (1)$$

$$\boldsymbol{y}_t = (\boldsymbol{\omega_0} - \boldsymbol{\omega_1} B - \boldsymbol{\omega_2} B^2)\boldsymbol{x}_{t-1} + \boldsymbol{n}_t \quad (2)$$

$\boldsymbol{y_t}$ is regressed with $\boldsymbol{x_t}$ according to the previous scenario until residual $\boldsymbol{u_t}$ is obtained. The parameter estimations use Equations (1) and (2) with OLS and GLS methods. $\boldsymbol{u_t}$ is formed into a GSTAR ($1_1$) model as in Equation (3).

$$\begin{bmatrix} u_{t,1} \\ u_{t,2} \\ u_{t,3} \end{bmatrix} = \left[\begin{pmatrix} \phi_{10} & 0 & 0 \\ 0 & \phi_{20} & 0 \\ 0 & 0 & \phi_{30} \end{pmatrix} + \begin{pmatrix} \phi_{11} & 0 & 0 \\ 0 & \phi_{21} & 0 \\ 0 & 0 & \phi_{31} \end{pmatrix} \begin{pmatrix} 0 & w_{12} & w_{13} \\ w_{21} & 0 & w_{23} \\ w_{31} & w_{32} & 0 \end{pmatrix}\right] \begin{pmatrix} u_1(t-1) \\ u_2(t-1) \\ u_3(t-1) \end{pmatrix} + \begin{pmatrix} \varepsilon_{t,1} \\ \varepsilon_{t,2} \\ \varepsilon_{t,3} \end{pmatrix} \quad (3)$$

parameter estimation in Equation (3) using OLS and GLS methods.

The efficiency of the estimation results of the GSTARX model using OLS is compared with the GLS based on the standard error (SE) coefficients $\omega_0$ and $\omega_1$ in Equations (1) and (2), and $\phi_{10}$, $\phi_{20}$, $\phi_{30}$, $\phi_{11}$, $\phi_{21}$, and $\phi_{31}$ in Equation (3).

Figure 1
Design of Data Generate $\boldsymbol{n_t}$[20]

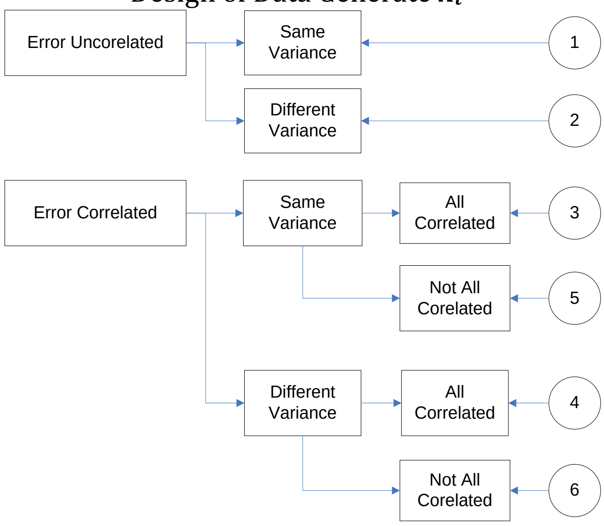


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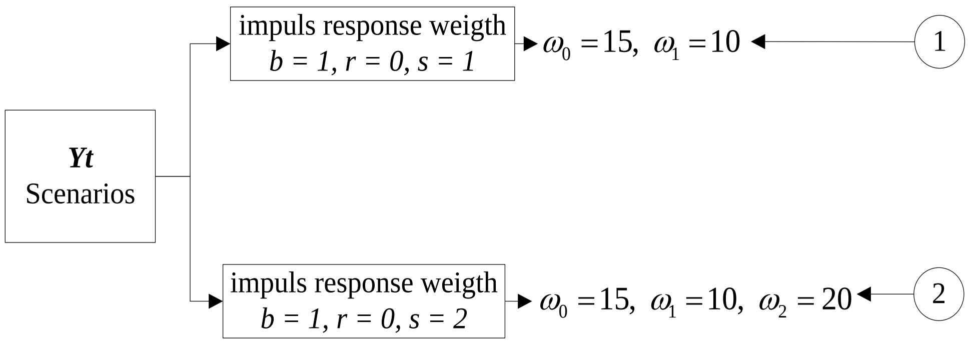
Figure 2
Scenarios of Data Generate $\boldsymbol{Y_t}$




## 2. Case Study

The data in the application were secondary inflow and outflow data in the City of Surabaya, Malang, Kediri, and Jember Regency from January 2003 to December 2014. The data were divided into in sample (training data from January 2003 to December 2011) and out sample (testing data from January 2012 to December 2014).

Table 1
Terms of Weekly Period

| Week | Date |
|---|---|
| 1 | 01, 02, 03, 04, 05, 06, and 07 |
| 2 | 08, 09, 10, 11, 12, 13, 14 and 15 |
| 3 | 16, 17, 18, 19, 20, 21, 22 and 23 |
| 4 | Non-Leap Year February;<br>24, 25, 26, 27 and 28<br>Leap Year February;<br>24, 25, 26, 27, 28 and 29<br>April, June, September and November;<br>24, 25, 26, 27, 28, 29 and 30<br>January, March, May, July, August, October,<br>and December;<br>24, 25, 26, 27, 28, 29, 30 and 31 |

Inflow data structure is divided into two predictors: dummy variables and metric variables. Dummy variables are used to express the calendar variations effects. It is a weekly period dummy for one month before and after Eid al-Fitr. Bank Indonesia Centre for Research and Education determines the weekly period in Table 1[21]. Based on these provisions, for inflow data modeling, Equation (4) is obtained.

[21] Legal Information Team, "Surat Edaran No.9/37/DPU Tanggal 27 Desember 2007 Tentang Penyetoran Dan Penarikan Uang Rupiah Oleh Bank Umum Di Bank Indonesia" (Bank Indonesia, 2007).

$$M_{i,t}\begin{cases}1, \text{ for t}^{\text{th}}\text{month w}\mathit{ith} \\ \quad \mathit{Eid\ al-Fitr\ Effect} \text{ in i}^{\text{th}} \text{ week} \\ 0, \text{ for another month}\end{cases}$$

$$M_{i,t+1}\begin{cases}1, \text{ for (t+1)}^{\text{th}} \text{ month w}\mathit{ith} \\ \quad \mathit{Eid\ al-Fitr\ Effect} \text{ in i}^{\text{th}} \text{ week} \\ 0, \text{ for another month}\end{cases} \tag{4}$$

Meanwhile, the outflow dummy is a weekly period dummy for one month before and after Eid al-Fitr. Based on these provisions, for outflow data modeling, Equation (5) is obtained.

$$M_{i,t}\begin{cases}1, \text{ for t}^{\text{th}}\text{month w}\mathit{ith} \\ \quad \mathit{Eid\ al-Fitr\ Effect} \text{ in i}^{\text{th}} \text{ week} \\ 0, \text{ for another month}\end{cases}$$

$$M_{i,t-1}\begin{cases}1, \text{ for (t-1)}^{\text{th}}\text{month w}\mathit{ith} \\ \quad \mathit{Eid\ al-Fitr\ Effect} \text{ in i}^{\text{th}} \text{ week} \\ 0, \text{ for another month}\end{cases} \tag{5}$$

The case study consists of two levels. Level 1 performs two parameter estimates of the model, namely the calendar variation model (for the Eid effect) and linear trend (for changes in Bank Indonesia regulations), and the transfer function model (for the CPI). OLS performed parameter estimation for these two models. After that, the two models are combined into one model, as shown in Equation (6) for inflow and Equation (7) for outflow. $\beta_1$, $\beta_2$, $\beta_3$, are $\beta_4$ are dummy variable coefficients for weeks 1, 2, 3, and 4 of Eid al-Fitr month. $\beta_5$, $\beta_6$, $\beta_7$, dan $\beta_8$ are the coefficients of dummy variables for weeks 1, 2, 3, and 4 of a month after (for Equation (6)) and before (for Equation (7)) Eid al-Fitr. $\gamma_1$, $\gamma_2$, $\gamma_3$ are for the first, second, and third trend intercepts, and $\gamma_4$, $\gamma_5$, $\gamma_6$ are the coefficients of the dummy variables for the first, second, and third trends.

$$y_t = \beta_1 M_{1,t} + \beta_2 M_{2,t} + \beta_3 M_{3,t} + \beta_4 M_{4,t} + \beta_5 M_{1,t+1} + \beta_6 M_{2,t+1} + \beta_7 M_{3,t+1} + \beta_8 M_{4,t+1} + \gamma_1 D_1 + \gamma_2 D_2 + \gamma_3 D_3 + \gamma_4 D_1 t_1 + \gamma_5 D_2 t_2 + \gamma_6 D_3 t_3 + v(B) x_t + u_t \tag{6}$$

$$y_t = \beta_1 M_{1,t} + \beta_2 M_{2,t} + \beta_3 M_{3,t} + \beta_4 M_{4,t} + \beta_5 M_{1,t-1} + \beta_6 M_{2,t-1} + \beta_7 M_{3,t-1} + \beta_8 M_{4,t-1} + \gamma_1 D_1 + \gamma_2 D_2 + \gamma_3 D_3 + \gamma_4 D_1 t_1 + \gamma_5 D_2 t_2 + \gamma_6 D_3 t_3 + v(B) x_t + u_t \tag{7}$$

Furthermore, in the level 2, the residuals from the combined model from the level 1 were formed into the GSTARX model. This model was estimated by the GLS parameter with four types of weights, namely uniform, distance inverse, normalized cross-correlation and normalized partial cross-correlation inference.

In-sample and out-sample forecasts were performed to compare the results of the GSTARX modelling from the four weights. The smallest RMSE value was the best GSTARX model.

## RESULTS AND DISCUSSION

This section presents the results and discussion of the GLS estimation and simulation, as well as the accuracy of forecasting of the GSTARX-GLS model with the effects of Eid al-Fitr and CPI.

### 1. Simulation Study Results

Results of the estimated efficiency of GLS against OLS at Levels 1 and 2 can be seen in Table 2 to Table 7. Equation (8) is the calculation formula used to determine the efficiency value in Table 2.

$$\textit{Efficiency}\ \left(SE(\hat{\beta})\right) = \frac{SE_{\mathrm{OLS}}(\hat{\beta}) - SE_{\mathrm{GLS}}(\hat{\beta})}{SE_{\mathrm{OLS}}(\hat{\beta})} \times 100\% \qquad (8)$$

First and second simulations generate relatively similar model parameter estimations. Its efficiency value is below 1%, which indicates that the OLS estimate is as good as the GLS estimate at Levels 1 and 2. This occurs because the locations are not correlated, so the OLS and GLS efficiency are relatively the same.

Table 2
The efficiency GLS against OLS in simulation 1

| Scenario | Location | Level 1 Parameter | Efficiency (%) | Level 2 Parameter | Efficiency (%) |
|---|---|---|---|---|---|
| 1 | 1 | $\hat{\omega}_0$ | 0,83% | $\hat{\phi}_{10}$ | 0,12% |
| | | $\hat{\omega}_1$ | 0,82% | $\hat{\phi}_{20}$ | 0,33% |
| | 2 | $\hat{\omega}_0$ | 1,14% | $\hat{\phi}_{30}$ | 0,42% |
| | | $\hat{\omega}_1$ | 1,13% | $\hat{\phi}_{11}$ | 0,07% |
| | 3 | $\hat{\omega}_0$ | 1,24% | $\hat{\phi}_{21}$ | 0,16% |
| | | $\hat{\omega}_1$ | 1,26% | $\hat{\phi}_{31}$ | 0,21% |
| 2 | 1 | $\hat{\omega}_0$ | 2,06% | $\hat{\phi}_{10}$ | 0,12% |
| | | $\hat{\omega}_1$ | 2,02% | $\hat{\phi}_{20}$ | 0,33% |
| | | $\hat{\omega}_2$ | 2,04% | | |
| | 2 | $\hat{\omega}_0$ | 2,43% | $\hat{\phi}_{30}$ | 0,42% |
| | | $\hat{\omega}_1$ | 2,40% | $\hat{\phi}_{11}$ | 0,07% |
| | | $\hat{\omega}_2$ | 2,42% | | |
| | 3 | $\hat{\omega}_0$ | 1,83% | $\hat{\phi}_{21}$ | 0,16% |
| | | $\hat{\omega}_1$ | 1,83% | $\hat{\phi}_{31}$ | 0,21% |
| | | $\hat{\omega}_2$ | 1,85% | | |

Table 3
The efficiency GLS against OLS in simulation 2

| Scenario | Location | Level 1 Parameter | Efficiency (%) | Level 2 Parameter | Efficiency (%) |
|---|---|---|---|---|---|
| 1 | 1 | $\hat{\omega}_0$ | 2,19% | $\hat{\phi}_{10}$ | 0,37% |
| | | $\hat{\omega}_1$ | 2,08% | $\hat{\phi}_{20}$ | 0,10% |
| | 2 | $\hat{\omega}_0$ | 0,54% | $\hat{\phi}_{30}$ | 0,45% |
| | | $\hat{\omega}_1$ | 0,55% | $\hat{\phi}_{11}$ | 0,22% |
| | 3 | $\hat{\omega}_0$ | 1,69% | $\hat{\phi}_{21}$ | 0,05% |
| | | $\hat{\omega}_1$ | 1,76% | $\hat{\phi}_{31}$ | 0,31% |
| 2 | 1 | $\hat{\omega}_0$ | 3,17% | $\hat{\phi}_{10}$ | 0,50% |
| | | $\hat{\omega}_1$ | 3,01% | $\hat{\phi}_{20}$ | 0,72% |
| | | $\hat{\omega}_2$ | 3,07% | | |
| | 2 | $\hat{\omega}_0$ | 3,47% | $\hat{\phi}_{30}$ | 0,27% |
| | | $\hat{\omega}_1$ | 3,39% | $\hat{\phi}_{11}$ | 0,29% |
| | | $\hat{\omega}_2$ | 3,56% | | |
| | 3 | $\hat{\omega}_0$ | 2,47% | $\hat{\phi}_{21}$ | 0,40% |
| | | $\hat{\omega}_1$ | 2,45% | $\hat{\phi}_{31}$ | 0,18% |
| | | $\hat{\omega}_2$ | 2,48% | | |

Table 4
The efficiency GLS against OLS in simulation 3

| Scenario | Location | Level 1 Parameter | Efficiency (%) | Level 2 Parameter | Efficiency (%) |
|---|---|---|---|---|---|
| 1 | 1 | $\hat{\omega}_0$ | 19,82% | $\hat{\phi}_{10}$ | 9,66% |
| | | $\hat{\omega}_1$ | 18,74% | $\hat{\phi}_{20}$ | 4,45% |
| | 2 | $\hat{\omega}_0$ | 7,64% | $\hat{\phi}_{30}$ | 9,09% |
| | | $\hat{\omega}_1$ | 7,70% | $\hat{\phi}_{11}$ | 7,33% |
| | 3 | $\hat{\omega}_0$ | 16,96% | $\hat{\phi}_{21}$ | 2,93% |
| | | $\hat{\omega}_1$ | 17,92% | $\hat{\phi}_{31}$ | 6,19% |
| 2 | 1 | $\hat{\omega}_0$ | 23,94% | $\hat{\phi}_{10}$ | 14,52% |
| | | $\hat{\omega}_1$ | 23,14% | $\hat{\phi}_{20}$ | 8,50% |
| | | $\hat{\omega}_2$ | 23,34% | | |
| | 2 | $\hat{\omega}_0$ | 15,18% | $\hat{\phi}_{30}$ | 9,08% |
| | | $\hat{\omega}_1$ | 14,76% | $\hat{\phi}_{11}$ | 11,80% |
| | | $\hat{\omega}_2$ | 15,11% | | |
| | 3 | $\hat{\omega}_0$ | 15,32% | $\hat{\phi}_{21}$ | 5,62% |
| | | $\hat{\omega}_1$ | 15,49% | $\hat{\phi}_{31}$ | 6,36% |
| | | $\hat{\omega}_2$ | 16,10% | | |

Table 5
The efficiency GLS against OLS in simulation 4

| Scenario | Location | Level 1 Parameter | Efficiency (%) | Level 2 Parameter | Efficiency (%) |
|---|---|---|---|---|---|
| 1 | 1 | $\hat{\omega}_0$ | 36,27% | $\hat{\phi}_{10}$ | 31.99% |
| | | $\hat{\omega}_1$ | 36,35% | $\hat{\phi}_{20}$ | 20.72% |
| | 2 | $\hat{\omega}_0$ | 19,18% | $\hat{\phi}_{30}$ | 21.35% |
| | | $\hat{\omega}_1$ | 19,17% | $\hat{\phi}_{11}$ | 28.02% |
| | 3 | $\hat{\omega}_0$ | 30,32% | $\hat{\phi}_{21}$ | 13.99% |
| | | $\hat{\omega}_1$ | 30,12% | $\hat{\phi}_{31}$ | 17.60% |
| 2 | 1 | $\hat{\omega}_0$ | 41,12% | $\hat{\phi}_{10}$ | 31.96% |
| | | $\hat{\omega}_1$ | 40,56% | $\hat{\phi}_{20}$ | 21.18% |
| | | $\hat{\omega}_2$ | 40,69% | | |
| | 2 | $\hat{\omega}_0$ | 27,43% | $\hat{\phi}_{30}$ | 17.92% |
| | | $\hat{\omega}_1$ | 27,51% | $\hat{\phi}_{11}$ | 28.16% |
| | | $\hat{\omega}_2$ | 27,54% | | |
| | 3 | $\hat{\omega}_0$ | 26,48% | $\hat{\phi}_{21}$ | 14.29% |
| | | $\hat{\omega}_1$ | 26,31% | $\hat{\phi}_{31}$ | 13.90% |
| | | $\hat{\omega}_2$ | 26,90% | | |

Table 6
The efficiency GLS against OLS in simulation 5

| Scenario | Location | Level 1 Parameter | Efficiency (%) | Level 2 Parameter | Efficiency (%) |
|---|---|---|---|---|---|
| 1 | 1 | $\hat{\omega}_0$ | 11,92% | $\hat{\phi}_{10}$ | 4,08% |
| | | $\hat{\omega}_1$ | 11,64% | $\hat{\phi}_{20}$ | 3,74% |
| | 2 | $\hat{\omega}_0$ | 11,52% | $\hat{\phi}_{30}$ | 8,63% |
| | | $\hat{\omega}_1$ | 11,57% | $\hat{\phi}_{11}$ | 2,58% |
| | 3 | $\hat{\omega}_0$ | 20,63% | $\hat{\phi}_{21}$ | 2,07% |
| | | $\hat{\omega}_1$ | 21,15% | $\hat{\phi}_{31}$ | 6,40% |
| 2 | 1 | $\hat{\omega}_0$ | 3,90% | $\hat{\phi}_{10}$ | 1,08% |
| | | $\hat{\omega}_1$ | 3,83% | $\hat{\phi}_{20}$ | 8,37% |
| | | $\hat{\omega}_2$ | 3,84% | | |
| | 2 | $\hat{\omega}_0$ | 13,15% | $\hat{\phi}_{30}$ | 9,86% |
| | | $\hat{\omega}_1$ | 12,84% | $\hat{\phi}_{11}$ | 0,62% |
| | | $\hat{\omega}_2$ | 12,84% | | |
| | 3 | $\hat{\omega}_0$ | 15,37% | $\hat{\phi}_{21}$ | 5,38% |
| | | $\hat{\omega}_1$ | 15,43% | $\hat{\phi}_{31}$ | 7,10% |

| | | | | | |
|---|---|---|---|---|---|
| | | $\hat{\omega}_2$ | 15,75% | | |

Table 7
The efficiency GLS against OLS in simulation 6

| Scenario | Location | Level 1 Parameter | Efficiency (%) | Level 2 Parameter | Efficiency (%) |
|---|---|---|---|---|---|
| 1 | 1 | $\hat{\omega}_0$ | 46,65% | $\hat{\phi}_{10}$ | 10,22% |
| | | $\hat{\omega}_1$ | 45,40% | $\hat{\phi}_{20}$ | 23,28% |
| | 2 | $\hat{\omega}_0$ | 58,53% | $\hat{\phi}_{30}$ | 70,28% |
| | | $\hat{\omega}_1$ | 58,50% | $\hat{\phi}_{11}$ | 4,70% |
| | 3 | $\hat{\omega}_0$ | 65,85% | $\hat{\phi}_{21}$ | 6,12% |
| | | $\hat{\omega}_1$ | 66,76% | $\hat{\phi}_{31}$ | 67,01% |
| 2 | 1 | $\hat{\omega}_0$ | 54,76% | $\hat{\phi}_{10}$ | 12,41% |
| | | $\hat{\omega}_1$ | 52,80% | $\hat{\phi}_{20}$ | 37,63% |
| | | $\hat{\omega}_2$ | 52,79% | | |
| | 2 | $\hat{\omega}_0$ | 67,20% | $\hat{\phi}_{30}$ | 73,51% |
| | | $\hat{\omega}_1$ | 66,98% | $\hat{\phi}_{11}$ | 6,62% |
| | | $\hat{\omega}_2$ | 67,74% | | |
| | 3 | $\hat{\omega}_0$ | 73,97% | $\hat{\phi}_{21}$ | 26,89% |
| | | $\hat{\omega}_1$ | 73,91% | | |
| | | $\hat{\omega}_2$ | 74,58% | $\hat{\phi}_{31}$ | 70,81% |

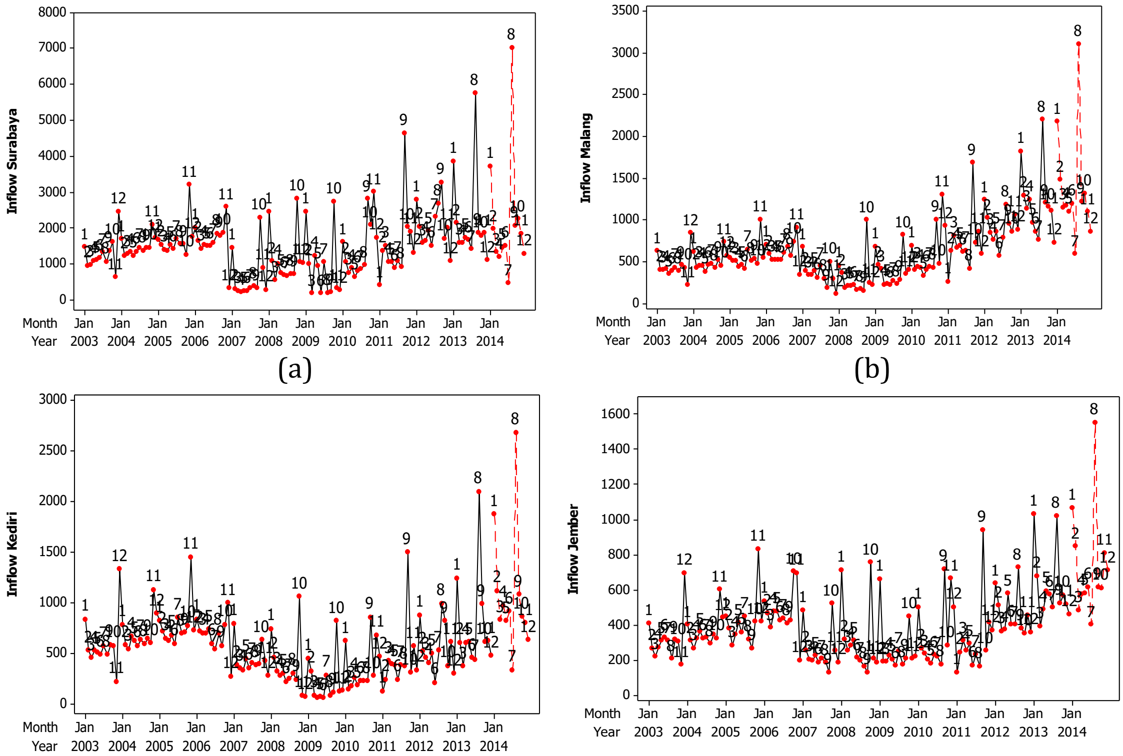
Meanwhile, simulations 3, 4, 5, and 6 produced efficiency values above 1%, with some reaching more than 70%. Therefore, it can be said that the estimation of GLS is better than OLS in these four simulations. This could be attributable to the location correlation, even though the scenarios are different. Therefore, in this study, only GLS estimation was carried out because the locations were assumed to be correlated based on inflow and outflow regulations issued by Bank Indonesia.[22]

## 2. Case Study Results

Figures 3 and 4 show that the City of Surabaya, Malang, Kediri, and Jember Regency have the same pattern every month for both inflow and outflow data. Every year the in-sample data always increases in a certain month. The same is true for the out-sample data, which shows a large increase in August and July for the outflows. This is because the determination of 1 Shawwal based on the Hijri calendar constantly changes yearly if it is adjusted to the Gregorian calendar. The dates of Eid al-Fitr can be seen in Table 8.

[22] Legal Information Team, "Surat Edaran Bank Indonesia Nomor 13/9/DPU Tanggal 5 April 2011 Perihal Penyetoran Dan Penarikan Uang Rupiah Oleh Bank Umum Di Bank Indonesia."

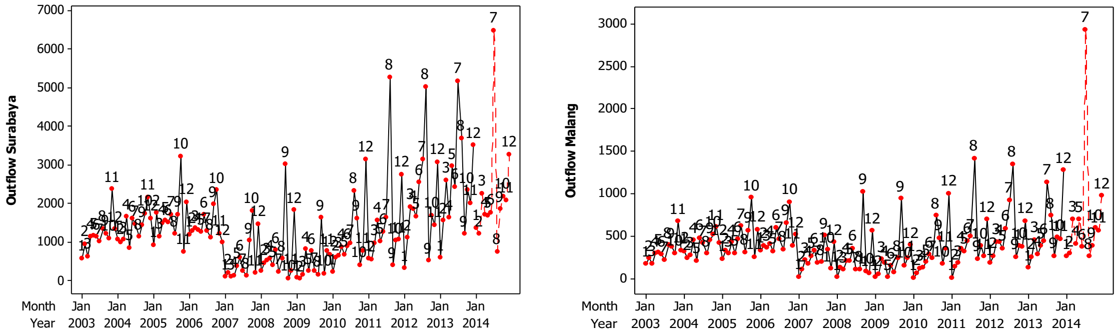
Figure 3
Inflow time series plots (a) Surabaya, (b) Malang, (c) Kediri, (d) Jember



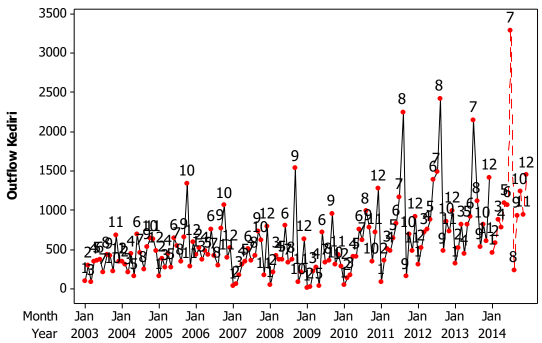

(a) (b)

(c) (d)

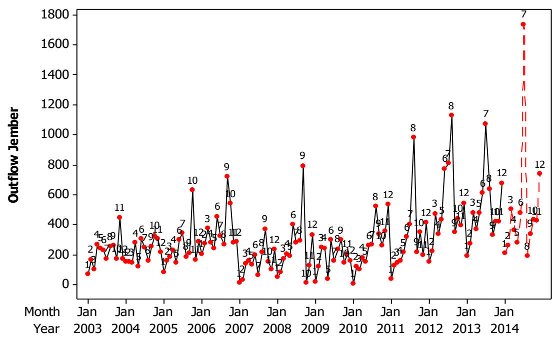
Figure 4
Outflow Time Series Plots (a) Surabaya, (b) Malang, (c) Kediri, (d) Jember




(a) (b)




(c)




(d)

Table 8
Dates of Eid Al-Fitr in 2003-2014

| Year | Date |
|---|---|
| 2003 | 25-26 November |
| 2004 | 14-15 November |
| 2005 | 03-04 November |
| 2006 | 23-24 October |
| 2007 | 12-13 October |
| 2008 | 01-02 October |
| 2009 | 21-22 September |
| 2010 | 10-11 September |
| 2011 | 30-31 August |
| 2012 | 19-20 August |
| 2013 | 08-09 August |
| 2014 | 28-29 July |

In addition, Figure 3 and Figure 4 show that there are three upward trends. The first occurred from January 2003 to December 2006, the second from January 2007 to December 2010, and the third from January 2011 to December 2013. The three trends of the inflow and outflow data were caused by regulations from Bank Indonesia. The first and second trends occurred due to the implementation of a trial policy on bank payment deposits, which was enforced from 28 October 2005, which was then regulated following Circular Letter No.9/37/DPU dated 27 December 2007 regarding Deposits and Withdrawals of Rupiah by Commercial Banks at Bank Indonesia[23]. This policy has been in effect nationally since December 2006 in all Bank Indonesia office areas, resulting in a significant decrease in currency flow in early 2007. Meanwhile, the second and third trends occurred because Bank Indonesia changed the mechanism for depositing and withdrawing the Rupiah by commercial banks at Bank Indonesia, as stated in Circular No.13/9/DPU in 2011.[24]

[23] Legal Information Team, "Surat Edaran No.9/37/DPU Tanggal 27 Desember 2007 Tentang Penyetoran Dan Penarikan Uang Rupiah Oleh Bank Umum Di Bank Indonesia."

[24] Legal Information Team, "Surat Edaran Bank Indonesia Nomor 13/9/DPU Tanggal 5 April 2011 Perihal Penyetoran Dan Penarikan Uang Rupiah Oleh Bank Umum Di Bank Indonesia."

### 2.1. Model Identification at Level 1

At this stage, two types of models have formed: the calendar variation model and the linear trend, as well as the transfer function model. In the calendar variation model and linear trend, linear regression was carried out at the same and different locations so that the $\varepsilon_i(t)$ model is obtained, with $i$ = 1, 2, 3, 4. Meanwhile, cross-correlation (CCF) plots are performed in the transfer function model between the CPI, which had been stationary in the mean, and the variance in each inflow and outflow. The CPI in the four locations was transformed quadratically to be stationary in the variance. Likewise, in overcoming the stationary in the mean, the CPI in the four locations was differentiated at lag 1.

After obtaining the two models, the significance of the combined model parameters was estimated and tested for each inflow and outflow in the four locations.

### 2.2. Estimation Parameters Model at Level 1

The combined model of the calendar variation model and linear trend, as well as the transfer function model, is as follows. The model formed is restricted with only significant parameters being included.

i. Inflow Model at Level 1:

- Surabaya:

$$y_1(t) = y_1(t-1) + 1901{,}1M_{1,t} + 1936{,}7M_{2,t} + 1416{,}8M_{3,t+1} + 2223{,}2M_{4,t+1} + 1350{,}2D_1 + 1982{,}2D_2 + 12{,}42D_2t_2 + (1{,}10\text{-}0{,}99B)x_{t-1}^2 + u_{1,t}$$

- Malang:

$$y_2(t) = y_2(t-1) + 535{,}70M_{1,t} + 501{,}16M_{2,t} + 756{,}97M_{4,t+1} + 358{,}93D_1 + 5{,}12D_1t_1 + 5{,}61D_2t_2 + 8{,}64D_3t_3 + (0{,}30\text{-}0{,}27B)x_{t-1}^2 + u_{2,t}$$

- Kediri:

$$y_3(t) = y_3(t-1) + 792{,}71M_{1,t} + 708{,}70M_{2,t} + 370{,}27M_{3,t+1} + 949{,}14M_{4,t+1} + 622{,}31D_1 + 300{,}69D_2 + 5{,}05D_3t_3 + (0{,}20 - 0{,}30B + 0{,}18B^{11})x_{t-1}^2 + u_{3,t}$$

- Jember:

$$y_4(t) = y_4(t-1) + 452{,}92M_{1,t} + 352{,}91M_{2,t} + 531{,}28M_{4,t+1} + 244{,}03D_1 + 4{,}81D_1t_1 + 4{,}33D_3t_3 + (0{,}14 - 0{,}18B)x_{t-1}^2 + u_{4,t}$$

ii. Outflow Model at Level 1:

- Surabaya:

$$y_1(t) = y_1(t-1) + 1175{,}1M_{2,t} + 1706{,}8M_{3,t} + 2627{,}4M_{4,t} + 2286{,}5M_{1,t-1} + 1441{,}7M_{2,t-1} + 996{,}73M_{3,t-1} + 1179{,}4D_1 + 965{,}71D_2 + 20{,}90D_2t_2 + 16{,}97D_3t_3 + (1{,}00\text{-}1{,}30B)x_t^2 + u_{1,t}$$

- Malang:

$$y_2(t) = y_2(t-1) + 24{,}12M_{2,t} + 695{,}98M_{3,t} + 745{,}98M_{4,t} + 699{,}47M_{1,t-1} + 413{,}55M_{2,t-1} + 365{,}28M_{3,t-1} + 244{,}98D_1 + 3{,}78D_1t_1 + 2{,}97D_2t_2 + 3{,}76D_3t_3 + (0{,}22 - 0{,}24B)x_t^2 + u_{2,t}$$

- Kediri:

$$y_3(t) = y_3(t-1) + 285{,}24M_{2,t} + 960{,}49M_{3,t} + 1017{,}9M_{4,t} + 1079{,}7M_{1,t-1} + 629{,}25M_{2,t-1} + 562{,}66M_{3,t-1} + 349{,}67D_1 + 5{,}42D_2t_2 + 6{,}77D_3t_3 + (0{,}33 - 0{,}39B)x_t^2 + u_{3,t}$$

- Jember:

$$y_4(t) = y_4(t-1) + 1706{,}8M_{3,t} + 2627{,}4M_{4,t} + 2286{,}5M_{1,t-1} + 1441{,}7M_{2,t-1} + 996{,}73M_{3,t-1} + 1179{,}4D_1 + 965{,}71D_1t_1 + 20{,}90D_2t_2 + 16{,}97D_3t_3 + (1{,}00 - 1{,}30B)x_t^2 + u_{4,t}$$

Residual $u_{i,t}$ with i = 1,2,3,4 from the model estimation results in each inflow and outflow model's response variable at Level 2.

### 2.3. Model Order Identification at Level 2

Residual $u_{i,t}$ was identified using the VAR model, so the MCCF and MPCCF plots were built, as shown in Figures 5 and 6.

Figure 5
Schematic representation of (a) MCCF and (b) MPCCF for inflows in four locations

```
Variable/
Lag      0    1    2    3    4    5    6    7
u1       ++++ ---- .... +.+. .... .... .... ....
u2       ++++ ---- .... .... .... .... .... ....
u3       ++++ ---- .... .... .-.. .... .... ....
u4       ++++ ---- .... +.+. .-.. .... .... ...-
```

(a)

```
Variable/
Lag      1    2    3    4    5    6    7    8
u1       -+.. -+.. .+.. .... .... .... .... ....
u2       --.. .-.. -.-. .... .... .... .... ....
u3       ..-+ ..-. .+-. .... .... .... .... ....
u4       ..-- -..- .... ...- ..+. .... .... ....
```

(b)

Figure 6
Schematic representation of (a) MCCF and (b) MPCCF for outflows in four locations

```
Variable/
Lag      0    1    2    3    4    5    6    7
u1       ++++ ---- .... .... .... .... .... ....
u2       ++++ ---- .... .... .... .... .... ....
u3       ++++ ---- .... .... .... .... .... ....
u4       ++++ ---- .... -... .... ...- .... ....
```

```
Variable/
Lag      1    2    3    4    5    6    7    8
u1       -.-. .... --.. .... +... .... ..+. ....
u2       .-.. .... .... .... .... .... .... ....
u3       ..-. ..-. .+.. -... -... .+.. .... ....
u4       ...- ...- .... .... .... .... .... ..-.
```

(a) (b)

Figures 5 (a) and (b) show that the VAR model for inflow is VAR (3). Meanwhile, Figures 6 (a) and (b) show that the VAR model for outflow data is VAR ([1,3]). Therefore, the time order of the GSTARX model on the inflow data is GSTARX ($3_1$), and the outflow data is GSTARX ([1,3] $_1$). Equation (9) is the GSTARX model ($3_1$) and Equation (10) is the GSTARX model ($[1,3]_1$).

$$u_{i,t}=\left[\boldsymbol{\Phi}_{10}^{1}+\boldsymbol{\Phi}_{11}\boldsymbol{W}^{1}\right]u_{i,(t-1)}+\left[\boldsymbol{\Phi}_{10}^{2}+\boldsymbol{\Phi}_{21}\boldsymbol{W}^{1}\right]u_{i,(t-2)}+\left[\boldsymbol{\Phi}_{10}^{3}+\boldsymbol{\Phi}_{31}\boldsymbol{W}^{1}\right]u_{i,(t-3)}+e_{i,t}$$

$$\begin{pmatrix}u_{1,t}\\u_{2,t}\\u_{3,t}\\u_{4,t}\end{pmatrix}=\left[\begin{pmatrix}\phi_{10}^{1}&0&0&0\\0&\phi_{20}^{1}&0&0\\0&0&\phi_{30}^{1}&0\\0&0&0&\phi_{40}^{1}\end{pmatrix}+\begin{pmatrix}\phi_{11}^{1}&0&0&0\\0&\phi_{21}^{1}&0&0\\0&0&\phi_{31}^{1}&0\\0&0&0&\phi_{41}^{1}\end{pmatrix}\begin{pmatrix}0&\omega_{12}&\omega_{13}&\omega_{14}\\\omega_{21}&0&\omega_{23}&\omega_{24}\\\omega_{31}&\omega_{32}&0&\omega_{34}\\\omega_{41}&\omega_{42}&\omega_{43}&0\end{pmatrix}\right]\begin{pmatrix}u_{1,(t-1)}\\u_{2,(t-1)}\\u_{3,(t-1)}\\u_{4,(t-1)}\end{pmatrix}+$$

$$\left[\begin{pmatrix}\phi_{10}^{2}&0&0&0\\0&\phi_{20}^{2}&0&0\\0&0&\phi_{30}^{2}&0\\0&0&0&\phi_{40}^{2}\end{pmatrix}+\begin{pmatrix}\phi_{11}^{2}&0&0&0\\0&\phi_{21}^{2}&0&0\\0&0&\phi_{31}^{2}&0\\0&0&0&\phi_{41}^{2}\end{pmatrix}\begin{pmatrix}0&\omega_{12}&\omega_{13}&\omega_{14}\\\omega_{21}&0&\omega_{23}&\omega_{24}\\\omega_{31}&\omega_{32}&0&\omega_{34}\\\omega_{41}&\omega_{42}&\omega_{43}&0\end{pmatrix}\right]\begin{pmatrix}u_{1,(t-2)}\\u_{2,(t-2)}\\u_{3,(t-2)}\\u_{4,(t-2)}\end{pmatrix}+$$

$$\left[\begin{pmatrix}\phi_{10}^{3}&0&0&0\\0&\phi_{20}^{3}&0&0\\0&0&\phi_{30}^{3}&0\\0&0&0&\phi_{40}^{3}\end{pmatrix}+\begin{pmatrix}\phi_{11}^{3}&0&0&0\\0&\phi_{21}^{3}&0&0\\0&0&\phi_{31}^{3}&0\\0&0&0&\phi_{41}^{3}\end{pmatrix}\begin{pmatrix}0&\omega_{12}&\omega_{13}&\omega_{14}\\\omega_{21}&0&\omega_{23}&\omega_{24}\\\omega_{31}&\omega_{32}&0&\omega_{34}\\\omega_{41}&\omega_{42}&\omega_{43}&0\end{pmatrix}\right]\begin{pmatrix}u_{1,(t-3)}\\u_{2,(t-3)}\\u_{3,(t-3)}\\u_{4,(t-3)}\end{pmatrix}+\begin{pmatrix}e_{1,t}\\e_{2,t}\\e_{3,t}\\e_{4,t}\end{pmatrix} \quad (9)$$

$$u_{i,t}=\left[\boldsymbol{\Phi}_{10}^{1}+\boldsymbol{\Phi}_{11}\boldsymbol{W}^{1}\right]u_{i,(t-1)}+\left[\boldsymbol{\Phi}_{10}^{3}+\boldsymbol{\Phi}_{31}\boldsymbol{W}^{1}\right]u_{i,(t-3)}+e_{i,t}$$

$$\begin{pmatrix}u_{1,t}\\u_{2,t}\\u_{3,t}\\u_{4,t}\end{pmatrix}=\left[\begin{pmatrix}\phi_{10}^{1}&0&0&0\\0&\phi_{20}^{1}&0&0\\0&0&\phi_{30}^{1}&0\\0&0&0&\phi_{40}^{1}\end{pmatrix}+\begin{pmatrix}\phi_{11}^{1}&0&0&0\\0&\phi_{21}^{1}&0&0\\0&0&\phi_{31}^{1}&0\\0&0&0&\phi_{41}^{1}\end{pmatrix}\begin{pmatrix}0&\omega_{12}&\omega_{13}&\omega_{14}\\\omega_{21}&0&\omega_{23}&\omega_{24}\\\omega_{31}&\omega_{32}&0&\omega_{34}\\\omega_{41}&\omega_{42}&\omega_{43}&0\end{pmatrix}\right]\begin{pmatrix}u_{1,(t-1)}\\u_{2,(t-1)}\\u_{3,(t-1)}\\u_{4,(t-1)}\end{pmatrix}+$$

$$\left[\begin{pmatrix}\phi_{10}^{3}&0&0&0\\0&\phi_{20}^{3}&0&0\\0&0&\phi_{30}^{3}&0\\0&0&0&\phi_{40}^{3}\end{pmatrix}+\begin{pmatrix}\phi_{11}^{3}&0&0&0\\0&\phi_{21}^{3}&0&0\\0&0&\phi_{31}^{3}&0\\0&0&0&\phi_{41}^{3}\end{pmatrix}\begin{pmatrix}0&\omega_{12}&\omega_{13}&\omega_{14}\\\omega_{21}&0&\omega_{23}&\omega_{24}\\\omega_{31}&\omega_{32}&0&\omega_{34}\\\omega_{41}&\omega_{42}&\omega_{43}&0\end{pmatrix}\right]\begin{pmatrix}u_{1,(t-3)}\\u_{2,(t-3)}\\u_{3,(t-3)}\\u_{4,(t-3)}\end{pmatrix}+\begin{pmatrix}e_{1,t}\\e_{2,t}\\e_{3,t}\\e_{4,t}\end{pmatrix} \quad (10)$$

## 2.4. Comparison of the Goodness of the GSTAR Model

This section uses weights of uniform, distance inverse, normalized cross-correlation and normalized cross-correlation inference. Especially for the GSTARX outflow data modelling for normalized cross-correlation inference, the partial value of cross-correlation among the locations is not valid, so the weights used are the same as for GSTARX modelling using uniform weight. Table 9 is the GSTARX in-sample RMSE values for inflow and outflow data, and Table 10 is GSTARX out-sample RMSE values for inflow and outflow data.

Table 9
RMSE Value of In-Sample Model GSTARX Data on Inflow and Outflow

| Model | Weight | City/Regency | | | | RMSE |
|---|---|---|---|---|---|---|
| | | Surabaya | Malang | Kediri | Jember | |
| Two-level GSTARX on the inflow | Uniform | 623,0 | 216,9 | 208,5 | 145,1 | 298,4 |
| | Distance inverse | 623,2 | 216,9 | 208,5 | 145,6 | 298,6 |
| | Cross-correlation normalization | 615,9 | 216,9 | 206,5 | 145,6 | 296,2 |
| | Partial cross-correlation inferential normalization | 614,3 | 224,4 | 209,3 | 145,1 | 298,3 |
| Two-level GSTARX on the outflow | Uniform | 717,0 | 198,2 | 307,0 | 145,8 | 408,9 |
| | Distance inverse | 717,7 | 198,2 | 307,0 | 145,8 | 409,2 |
| | Cross-correlation normalization | 715,6 | 198,0 | 307,0 | 145,8 | 341,6 |

The smallest RMSE, as shown in Table 9 in the GSTARX in-sample data modeling with two levels in the inflow, is found in the partial cross-correlation inference normalization in Jember Regency. Likewise, the city of Surabaya is well modeled using this weight. Meanwhile, Malang City and Kediri City are well modeled by the two-level GSTARX with cross-correlation normalization weights.

Meanwhile, the smallest RMSE, as shown in Table 9 in the GSTARX in-sample data modeling with two levels in the outflow, is found in the cross-correlation inference normalization in Jember Regency. Likewise, the city of Surabaya is well modeled using this weight. By contrast, Malang and Kediri are well modeled by the two-level GSTARX with cross-correlation normalization weights.

Table 10
RMSE Value of Out-Sample Model GSTARX Data on Inflow and Outflow

| Model | Weight | City/ Regency | | | | RMSE |
|---|---|---|---|---|---|---|
| | | Surabaya | Malang | Kediri | Jember | |
| Two-level GSTARX on the inflow | Uniform | 889,3 | 507,9 | 543,4 | 165,2 | 526,5 |
| | Distance inverse | 937,1 | 526,9 | 553,4 | 175,0 | 548,1 |
| | Cross-correlation normalization | 879,4 | 508,1 | 535,5 | 165,3 | 522,1 |
| | Partial cross-correlation inference normalization | 876,5 | 497,7 | 536,4 | 165,3 | 519,0 |

| Model | Weight | City/ Regency | | | | RMSE |
|---|---|---|---|---|---|---|
| | | Surabaya | Malang | Kediri | Jember | |
| Two-level GSTARX on the outflow | Uniform | 682,1 | 448,9 | 367,6 | 224,4 | 430,7 |
| | Distance inverse | 681,4 | 448,2 | 367,6 | 224,4 | 430,4 |
| | Cross-correlation normalization | 683,2 | 449,0 | 368,0 | 224,4 | 431,2 |

The smallest RMSE, as shown in Table 10 in the GSTARX out-sample data modeling with two levels in the inflow, is found in the uniform weight in Jember Regency. Meanwhile, Surabaya and Malang are well modeled using partial cross-correlation inference normalization, and Kediri using cross-correlation normalization.

The smallest RMSE, as shown in Table 10 in the GSTARX out-sample data modeling with two levels in the outflow, is found in the cross-correlation normalization in Jember Regency. Meanwhile, Surabaya, Malang, and Kediri are well modeled using distance inverse weight.

Overall, the best model obtained is the two-level GSTARX model with cross-correlation normalization. Equation (11) is this model for inflow, and Equation (12) is the model for outflow.

$$\begin{pmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t} \end{pmatrix} = \left[ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -0.42 & 0 & 0 \\ 0 & 0 & -0.53 & 0 \\ 0 & 0 & 0 & -0.47 \end{pmatrix} + \begin{pmatrix} 1.04 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0.37 & 0.34 & 0.29 \\ 0.24 & 0 & 0.37 & 0.38 \\ 0.27 & 0.37 & 0 & 0.36 \\ 0.29 & 0.41 & 0.30 & 0 \end{pmatrix} \right] \begin{pmatrix} u_{1,(t-1)} \\ u_{2,(t-1)} \\ u_{3,(t-1)} \\ u_{4,(t-1)} \end{pmatrix} +$$

$$\left[ \begin{pmatrix} -0.40 & 0 & 0 & 0 \\ 0 & -0.26 & 0 & 0 \\ 0 & 0 & -0.43 & 0 \\ 0 & 0 & 0 & -0.36 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0.32 & 0.22 & 0.47 \\ 0.43 & 0 & 0.31 & 0.26 \\ 0.37 & 0.35 & 0 & 0.28 \\ 0.35 & 0.36 & 0.29 & 0 \end{pmatrix} \right] \begin{pmatrix} u_{1,(t-2)} \\ u_{2,(t-2)} \\ u_{3,(t-2)} \\ u_{4,(t-2)} \end{pmatrix} +$$

$$\left[ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.27 & 0 \\ 0 & 0 & 0 & -0.11 \end{pmatrix} + \begin{pmatrix} -0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0.34 & 0.45 & 0.70 \\ 0.62 & 0 & 0.61 & 0.63 \\ 0.46 & 0.19 & 0 & 0.52 \\ 0.35 & 0.26 & 0.18 & 0 \end{pmatrix} \right] \begin{pmatrix} u_{1,(t-3)} \\ u_{2,(t-3)} \\ u_{3,(t-3)} \\ u_{4,(t-3)} \end{pmatrix} + \begin{pmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \\ e_{4,t} \end{pmatrix}. \tag{11}$$

The two-level GSTARX ($3_1$) model for $u_{i,t}$ inflow in Equation (17) can be written based on each location as follows:

i. GSTARX ($3_1$) Model with two-level inflow in Surabaya is

$$u_{1,t} = 0,37u_{2,(t-1)} + 0,34u_{3,(t-1)} + 0,30u_{4,(t-1)} - 0,40u_{1,(t-2)} - 0,87u_{2,(t-3)} - 0,11u_{3,(t-3)} + - 0,18u_{4,(t-3)} + e_{1,t}$$

ii. GSTARX ($3_1$) Model with two-level inflow in Malang is

$u_{2,t} = -0,42u_{2,(t-1)} - 0,26u_{2,(t-2)} + e_{2,t}$

iii. GSTARX ($3_1$) Model with two-level inflow in Kediri is
$u_{3,t} = -0,53u_{3,(t-1)} - 0,43u_{3,(t-2)} + 0,04u_{1,(t-3)} + 0,02u_{2,(t-3)} - 0,27u_{3,(t-3)} + 0,04u_{4,(t-3)} + e_{3,t}$

iv. GSTARX ($3_1$) Model with two-level inflow in Jember is
$u_{4,t} = -0,47u_{4,(t-1)} - 0,36u_{4,(t-2)} - 0,11u_{4,(t-3)} + e_{4,t}$

Based on the model formed for inflow in each location, it can be concluded that the effect of inflow in a location is influenced by the inflow in that location at different times. A location can also be affected by the inflow at different locations and at different times. For example, the inflow in Surabaya was influenced by the inflow in Surabaya itself in the previous two months (by -0.40) and in three other locations in a month (by 0.37 from Malang, 0.34 from Kediri, and 0.30 from Jember) and three months earlier (-0.87 from Malang, -0.11 from Kediri and -0.18 from Jember). Likewise, inflow in Malang, Kediri, and Jember has the same model interpretation as Surabaya.

$$\begin{pmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t} \end{pmatrix} = \left[ \begin{pmatrix} -0.36 & 0 & 0 & 0 \\ 0 & -0.35 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.32 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.34 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0.37 & 0.31 & 0.32 \\ 0.42 & 0 & 0.30 & 0.28 \\ 0.37 & 0.33 & 0 & 0.30 \\ 0.36 & 0.36 & 0.28 & 0 \end{pmatrix} \right] \begin{pmatrix} u_{1,(t-1)} \\ u_{2,(t-1)} \\ u_{3,(t-1)} \\ u_{4,(t-1)} \end{pmatrix} +$$

$$\left[ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} -0.41 & 0 & 0 & 0 \\ 0 & -0.04 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0.14 & 0.44 & 0.42 \\ 0.52 & 0 & 0.26 & 0.22 \\ 0.66 & 0.22 & 0 & 0.11 \\ 0.34 & 0.36 & 0.31 & 0 \end{pmatrix} \right] \begin{pmatrix} u_{1,(t-3)} \\ u_{2,(t-3)} \\ u_{3,(t-3)} \\ u_{4,(t-3)} \end{pmatrix} + \begin{pmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \\ e_{4,t} \end{pmatrix}. \tag{12}$$

The two-level GSTARX ([1,3]1) model for $u_{i,t}$ outflow in Equation (12) based on each location can be written as follows:

i. GSTARX ($[1,3]_1$) Model with two-level in Surabaya is
$u_{1,t} = -0,36u_{1,(t-1)} - 0,55u_{2,(t-3)} - 0,18u_{3,(t-3)} - 0,17u_{4,(t-3)} + e_{1,t}$

ii. GSTARX ($[1,3]_1$) Model with two-level in Malang is
$u_{2,t} = -0,35u_{2,(t-1)} - 0,02u_{1,(t-3)} - 0,01u_{3,(t-3)} - 0,01u_{4,(t-3)} + e_{2,t}$

iii. GSTARX ($[1,3]_1$) Model with two-level in Kediri is
$u_{3,t} = -0,12u_{1,(t-1)} - 0,11u_{2,(t-1)} - 0,10u_{4,(t-1)} + e_{3,t}$

iv. GSTARX ($[1,3]_1$) Model with two-level in Jember is
$u_{4,t} = -0,32u_{4,(t-1)} + e_{4,t}$

Based on the model formed for outflow in each location, it can be concluded that the effect of outflow in a location is influenced by the outflow in that location at different times. A location can also be affected by the outflow at different locations and at different times. For example, the outflow in Surabaya was influenced by the outflow in Surabaya itself in the previous month (by -0.36) and in three other locations in three months (by -0.55 from Malang, -0.18 from Kediri and -0.17 from Jember) and three months earlier (-0.87 from Malang, -0.11 from Kediri and -0.18 from Jember). Likewise, outflow in Malang, Kediri, and Jember has the same model interpretation as Surabaya.

## CONCLUSION AND SUGGESTIONS

The results of the simulation show the statistical values to identify VAR order. Time dependencies use MPCCF, while the effect order from exogenous variables uses the two-level GSTARX model with CCF at Level 1. The results also show that the two-level GSTARX estimator using GLS is more efficient than OLS, especially in simulating the correlated residuals between locations.

The effect of the Eid al-Fitr calendar variation on Surabaya and Kediri inflow data is if the holiday date is in the 1st and 2nd week. The effect also occurs one month after, if the date is in the 3rd and 4th week. In Malang and Jember, the effect of the Eid al-Fitr calendar variation occurs in that month if the holiday date is in the 1st and 2nd week. The effect also occurs one month after that if the holiday date is in the 4th week.

Furthermore, the outflow data in the cities of Surabaya, Malang, and Kediri experiences the effects of variations in the Eid al-Fitr calendar in that month if the holiday date is in the 2nd, 3rd, and 4th week. The effect also occurs one month earlier if the holiday dates are in the 1st and 2nd weeks. Meanwhile, outflow in Jember experiences the effect of Eid al-Fitr calendar variations in that month if the holiday date is in the 3rd and 4th week. It also occurs one month earlier if the holiday date is in the 1st and 3rd weeks.

Meanwhile, the two-level GSTARX-GLS model formed from the results of this study is GSTARX (31) for inflow dan GSTAR ([1,3]1) for outflow in four cities/districts in East Java. Generally, the best model obtained is a two-level GSTARX model with normalized cross-correlation.

In this study, suggestions that can be given for further research based on the results of the analysis and discussion are that the results may be better if the GLS estimation method is applied at level 1 to overcome the correlated residuals between locations. Then, it comes out during the analysis stage at level 1 or 2, that there is a seasonal pattern in various locations. Future research will also consider the MA order in the GSTARX model, also known as the GSTARIMAX model.

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