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TOWARD AN INTEGRATED TEACHER-KNOWLEDGE CONSTRUCT: SYNTHESIZING CONTENT, PEDAGOGICAL, AND CULTURALLY RESPONSIVE KNOWLEDGE FOR ELEMENTARY FRACTION TEACHING IN INDONESIA

Rafika Putri[1], Wiryanto[2*], Julien Chintya[3], Muhammad Rusli[4], Savira Hadiyanti[5]
Universitas Negeri Surabaya[1,2,4,5], Universitas Negeri Manado[3], Institut Cokroaminoto Pinrang[4]
rafikaaputrii@gmail.com[1], wiryanto@unesa.ac.id[2], julienchintya@unima.ac.id[3], muhammad.rusli0318@gmail.com[4], 25010855074@mhs.unesa.ac.id[5]*

Abstrak
K-CRMT (*Knowledge of Culturally Responsive Mathematics Teaching*) adalah pengetahuan guru sekolah dasar tentang cara mengajarkan matematika, khususnya pecahan, secara responsif terhadap latar belakang budaya peserta didik, termasuk di Madrasah Ibtidaiyah. Dua kerangka yang telah ada hanya mencakup sebagian dari pengetahuan ini: Mathematical Knowledge for Teaching mencakup konten dan pedagogi tetapi tidak budaya, sedangkan Culturally Responsive Mathematics Teaching mencakup budaya tetapi tidak konten atau pedagogi, dan belum ada kerangka yang mengintegrasikan keduanya untuk satu topik matematika. Artikel konseptual ini membangun integrasi tersebut: kedua kerangka ditelusuri ke tiga akar bersama dan digabungkan menjadi model tiga lingkaran, menghasilkan enam dimensi terukur dengan K-CRMT sebagai faktor orde kedua yang menjelaskan korelasi di antara dimensi tersebut, bukan sebagai dimensi ketujuh yang berdiri sendiri. Ini merupakan usulan teoretis yang hanya memenuhi bukti validitas berbasis konten, bukan instrumen yang telah tervalidasi; telaah pakar dan uji psikometrik direkomendasikan sebagai langkah berikutnya. Bagi pendidikan guru sekolah dasar dan Madrasah Ibtidaiyah, K-CRMT menawarkan satu kerangka eksplisit untuk merancang pembelajaran yang mengintegrasikan konten, pedagogi, dan responsivitas budaya secara bersamaan, bukan sebagai kompetensi-kompetensi terpisah.
Kata Kunci: K-CRMT, Validitas Konstruk, Inovasi Asesmen, Pembelajaran Matematika Responsif Budaya, Profesionalisme Guru Sekolah Dasar

Abstract
K-CRMT (Knowledge of Culturally Responsive Mathematics Teaching) is elementary teachers' knowledge of teaching mathematics, particularly fractions, responsively to students' cultural backgrounds, including in Madrasah Ibtidaiyah. Two existing frameworks cover only part of this: Mathematical Knowledge for Teaching addresses content and pedagogy but not culture, while Culturally Responsive Mathematics Teaching addresses culture but not content or pedagogy, and no framework yet integrates both for one topic. This conceptual article builds that integration: the two frameworks are traced to

three shared roots and recombined into a three-circle model, yielding six measurable dimensions with K-CRMT as the second-order factor explaining their correlation, not a seventh, parallel dimension. This theoretical proposal meets only content-based validity evidence, not a validated instrument; expert judgment and psychometric testing are recommended next. For elementary and Madrasah Ibtidaiyah teacher education, K-CRMT offers one explicit framework for designing instruction that integrates content, pedagogy, and cultural responsiveness, not as separate competencies.
Keywords: K-CRMT, Construct Validity, Assessment Innovation, Culturally Responsive Mathematics Teaching, Elementary Teacher Professionalism

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INTRODUCTION

Elementary school teachers in Indonesia, including those teaching in Madrasah Ibtidaiyah (MI), teach mathematics to students with highly diverse cultural backgrounds, languages, and social experiences. At the same time, the push toward globally oriented mathematics learning standards often treats mathematics as culturally neutral knowledge, even though the way students make sense of mathematical concepts, including fractions, is continually shaped by their cultural and linguistic context. Bartell, Drake, Roth-McDuffie, Aguirre, Turner, and Foote (2019) argue that mathematics teacher education needs validated frameworks and instruments that measure teacher knowledge in an integrated way, capturing mathematical content, pedagogy, and cultural awareness together rather than measuring each one separately.

This need is especially concrete for fraction teaching, where the mismatch between global curricular standards and local cultural knowledge is most visible in Indonesian elementary classrooms, including Madrasah Ibtidaiyah. Two recent studies illustrate this directly: Putri, Nurwidiyanto, and Rohmah (2020) developed an ethnomathematics-based fraction textbook for third-grade MI students and found it produced significantly greater learning gains than conventional instruction, while Noor, Ahsani, Ainurrokhimah, and Farid (2024) developed ethnomathematics-based mathematics teaching materials for MI students in Lombok and found them feasible and well received by students. Both studies show that grounding fraction instruction in local cultural practices, students' everyday mathematical experiences, and culturally situated representations of fractions, for example partitioning harvested produce or woven cloth, can work in the MI classroom specifically. What neither study addresses, however, is the teacher's own knowledge: both develop culturally responsive materials for students to use, not a framework for what a teacher needs to know in order to design or adapt such materials in the first place, including for the specific characteristics of mathematics learning at the MI level, where instruction sits

within a broader Islamic education curriculum alongside the national mathematics curriculum. This is the knowledge-level gap this article addresses. Two major theoretical strands have developed to address this teacher-knowledge gap, each reviewed in turn below before this article identifies the gap between them.

Mathematical Knowledge for Teaching (MKT), developed by Ball, Thames, and Phelps (2008), details the content and pedagogical knowledge domains a mathematics teacher needs to translate subject matter into instruction, building on Shulman's (1986, 1987) foundational distinction between content knowledge and pedagogical content knowledge. MKT already combines what this article later treats as two separate first-order dimensions, namely command of the mathematical content itself and the general pedagogical skill needed to teach it, into a single, content-and-pedagogy-specific construct (Ball et al., 2008; Hill, Rowan, & Ball, 2005). As a teacher-knowledge framework, MKT provides strong coverage of the content and pedagogy dimensions of mathematics teaching, but on its own it does not specify how a teacher should attend to students' cultural backgrounds, which is the dimension the second strand below addresses.

Subsequent empirical work substantiates MKT's core claim that mathematical knowledge for teaching is distinct from ordinary content mastery and does matter for student outcomes. Hill, Rowan, and Ball (2005) found a measurable, significant association between teachers' MKT scores and their students' mathematics achievement gains, net of other teacher characteristics, providing some of the first large-scale empirical support for treating MKT as a genuine competency rather than an abstract theoretical distinction. Baumert et al. (2010) extended this finding using German secondary mathematics teachers, showing that teachers' content knowledge predicted their pedagogical content knowledge, and that PCK, more than content knowledge alone, explained the cognitive demand of classroom instruction and, through it, student progress. Hill, Ball, and Schilling (2008) further unpacked PCK into several interrelated subdomains, including knowledge of content and students and knowledge of content and teaching, a decomposition this article revisits later when arguing that MKT itself, and not only PCK, can be split into separable components. Depaepe, Verschaffel, and Kelchtermans (2013), in a systematic review of how PCK has been used in mathematics education research, caution that the concept has been operationalized inconsistently across studies, which strengthens the case made below for defining each K-CRMT dimension explicitly rather than borrowing an unexamined label from the literature.

Culturally Responsive Mathematics Teaching (CRMT), as formulated by Aguirre and Zavala (2013), emphasizes the cultural, linguistic, mathematical thinking, and social justice dimensions of mathematics instruction. CRMT traces back to a broader tradition of culturally responsive pedagogy: Ladson-Billings (1995) formulated culturally relevant pedagogy as practice that builds academic success, cultural competence, and critical consciousness in students at the same time, while Gay (2018) extended this into culturally responsive teaching, which positions students' cultural backgrounds as a learning resource rather than an obstacle. Within mathematics specifically, Moll, Amanti, Neff, and Gonzalez (1992) introduced the notion of funds of knowledge held by families and communities as a basis for planning instruction, an idea Civil (2007) applied directly to mathematics education as a path toward equity, and one Gutiérrez

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(2002) framed as an equity research agenda for mathematics teachers. CRMT in this article inherits that tradition by focusing it on the four dimensions of cultural responsiveness (Aguirre & Zavala, 2013) that are specifically relevant to fraction instruction in Indonesian elementary schools. Applied concretely to fraction teaching, these four dimensions mean recognizing the identities and languages students use to name and compare fraction quantities, drawing on mathematical ways of thinking already present in local practices such as dividing harvested produce or partitioning woven cloth (D'Ambrosio, 1985), and treating equitable access to fraction reasoning, including for students in Madrasah Ibtidaiyah and other faith-based elementary schools, as a matter of social justice rather than optional enrichment. As with MKT, CRMT is a well-developed construct on its own terms, but by design it does not specify the mathematical content or pedagogical skill dimensions that MKT covers.

Empirical research has since substantiated these theoretical claims about culturally responsive mathematics teaching in practice. Roth McDuffie, Foote, Bolson, Turner, Aguirre, Bartell, Drake, and Land (2014) found that structured video analysis helped prospective K-8 teachers learn to notice the multiple mathematical knowledge bases their students bring from home and community settings, a noticing skill CRMT treats as a prerequisite for responsive instruction rather than an automatic byproduct of good intentions. Parker, Bartell, and Novak (2017) extended this line of work, showing that secondary mathematics teachers' conceptions of what it means to know their students develop gradually over a multi-year professional trajectory rather than settling after a single training experience, which signals that CRMT is better treated as a developable competency with measurable growth than as a fixed disposition. Within the specific domain of ethnomathematics, D'Ambrosio (1985) documented how mathematical reasoning is embedded in the everyday cultural practices of distinct communities, a foundation Rosa and Orey (2013) later formalized into ethnomodelling as a research lens connecting community mathematical practices to classroom modelling activities. Taken together, this body of work establishes that CRMT, like MKT, is grounded in a mature and still-developing empirical literature, which is precisely why combining the two, rather than treating either as sufficient on its own, is the more defensible path toward a validated teacher-knowledge construct.

Rubel and McCloskey (2021) caution that combining these two frameworks mechanically, by simply adding MKT components to CRMT components, does not produce a coherent construct; a synthesis of the two needs to be built as a new, integrated construct rather than assembled after the fact. This issue has grown more urgent since Louie (2017) documented that a culture of exclusion in mathematics learning persists even in classrooms where teachers are explicitly equity-oriented, which points to good pedagogical intent alone not being enough without a clear, measurable teacher-knowledge framework. Luzano (2025) conducted a scoping review of the literature on culturally responsive mathematics teaching strategies and found that no validated instrument yet exists that explicitly integrates MKT and CRMT for a specific mathematics topic. This gap persists even though ethnomathematics research (D'Ambrosio, 1985) has long documented the richness of locally grounded mathematical practice, since that documentation has not yet been translated into a psychometrically measurable instrument for teacher competence. Recent developments suggest that the push toward this kind of teacher-knowledge integration is gaining strength: Ogodo (2024) proposed a Culturally Responsive

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Pedagogical Knowledge framework that unites three STEM teacher-knowledge constructs under one conceptual umbrella, while Sarkar et al. (2024) confirmed through a systematic review that pedagogical content knowledge itself remains a construct still under development, with its interaction with other knowledge domains not yet fully mapped. On the classroom-practice side, Stone, Smith, and Ebner (2025) along with Comstock et al. (2025) show that CRMT research over the past five years has stayed centered on curriculum-material analysis and teaching practice rather than on a psychometrically integrated teacher-knowledge instrument for a specific mathematics topic such as fractions.

This study selects fractions as its application context because fractions are widely recognized as one of the most cognitively challenging elementary mathematics concepts for students and one of the most pedagogically complex for teachers (Kieren, 1976; Behr, Lesh, Post, & Silver, 1983), a difficulty that recent cognitive research confirms persists across student populations (Siegler, Fazio, Bailey, & Zhou, 2013) and remains one of the most persistent mathematical obstacles across school levels (Lortie-Forgues, Tian, & Siegler, 2015), making it a fitting proving ground for integrating content-pedagogical knowledge with cultural awareness. Building on this gap, this article's objective is to construct a content-based construct validity argument, Standard 1 of the six Standards in the Standards for Educational and Psychological Testing (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education [AERA, APA, & NCME], 2014), for Knowledge of Culturally Responsive Mathematics Teaching (K-CRMT) as a proposed second-order construct that integrates Mathematics Content Knowledge, Pedagogical Content Knowledge, and CRMT for fraction teaching. The article's contribution is this explicit construct definition and its positioning relative to MKT and CRMT, not a validated instrument: it is a conceptual contribution, and the claims made below should be read as a theoretical argument awaiting empirical validation, not as established findings.

METHOD

This article follows a conceptual research design, specifically the Theory Synthesis approach for conceptual articles described by Jaakkola (2020), in which multiple established theoretical perspectives are integrated into a single, more comprehensive framework by identifying the structure underlying them, rather than through the collection and testing of new empirical data. Theory Synthesis was chosen over an integrative review or a systematic/scoping review because the goal here is not to summarize or appraise the existing empirical literature on teacher knowledge, which those review types are designed to do, but to build a new theoretical structure by tracing established frameworks back to shared conceptual roots and reassembling them, which is the specific purpose Theory Synthesis serves among conceptual-article approaches (Jaakkola, 2020). No primary data were collected at this stage: no teachers, students, or schools served as a sample. The data sources are published theoretical and empirical literature on Pedagogical Content Knowledge (Shulman, 1986, 1987; Hill, Ball, & Schilling, 2008; Depaepe, Verschaffel, & Kelchtermans, 2013), Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005; Baumert et al., 2010), other integrated teacher-knowledge frameworks (Mishra & Koehler, 2006), Culturally Responsive Mathematics Teaching

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(Aguirre & Zavala, 2013), culturally responsive pedagogy more broadly (Ladson-Billings, 1995; Gay, 2018), family and community funds of knowledge in mathematics learning (Moll, Amanti, Neff, & Gonzalez, 1992; Civil, 2007; Gutiérrez, 2002), ethnomathematics (D'Ambrosio, 1985; Rosa & Orey, 2013), the cognitive characteristics of learning fractions (Kieren, 1976; Behr, Lesh, Post, & Silver, 1983; Siegler, Fazio, Bailey, & Zhou, 2013; Lortie-Forgues, Tian, & Siegler, 2015), construct validity theory (Messick, 1995; Kane, 2013), and the Standards for Educational and Psychological Testing framework (AERA, APA, & NCME, 2014). Literature was included in this synthesis under three criteria: it originated one of the three source frameworks (MKT, Shulman's foundational PCK work, or CRMT) or directly extended one of them theoretically or empirically; it appeared in a peer-reviewed journal or academic book; and, for the cognitive and fraction-specific literature, it addressed fraction learning or teaching specifically rather than mathematics learning in general. Literature outside these criteria, including the two Madrasah Ibtidaiyah studies discussed in the Introduction (Noor et al., 2024; Putri, Nurwidiyanto, & Rohmah, 2020), was used only to motivate the problem and was not treated as a source for deriving the six K-CRMT dimensions themselves, since neither study addresses teacher knowledge directly.

The analysis proceeds in three stages: first, identifying the core theoretical propositions of Mathematics Content Knowledge, Pedagogical Content Knowledge, and CRMT relevant to fraction instruction; second, mapping these propositions onto a three-circle model that yields six first-order dimensions along with one second-order factor (K-CRMT); and third, building a construct validity argument for Standard 1 (content-based evidence) through construct definition and a construct map. Table 1 makes this three-stage process explicit by tracing each of the three source frameworks through its core theoretical contribution, its own stated limitation, and what it specifically contributes to the K-CRMT construct derived in the Results section.

Table 1 - Synthesis of Source Frameworks into the K-CRMT Construct

Theoretical Framework	Core Contribution	Stated Limitation	Contribution to K-CRMT
Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008)	Integrates mathematical content mastery and the pedagogical skill to teach it	Does not specify how a teacher should attend to students' cultural backgrounds	Decomposed into the Mathematics Content Knowledge and Pedagogical Content Knowledge pure circles, and retained as the content-pedagogy (MKT) intersection dimension
Pedagogical Content Knowledge (Shulman, 1986, 1987)	Establishes general pedagogical skill for teaching content effectively, distinct from content mastery itself	Not specific to mathematics or to any cultural context	Supplies the Pedagogical Content Knowledge pure-circle definition

Theoretical Framework	Core Contribution	Stated Limitation	Contribution to K-CRMT
Culturally Responsive Mathematics Teaching (Aguirre & Zavala, 2013)	Specifies four dimensions of cultural responsiveness in mathematics instruction (identity, language, mathematical thinking, social justice)	Does not specify the mathematical content or pedagogical skill a teacher needs	Supplies the Culturally Responsive Mathematics Teaching pure circle
Ethnomathematics (D'Ambrosio, 1985)	Documents mathematical reasoning embedded in everyday cultural practice	Documents cultural-mathematical practice but not how to teach it	Supplies the content-culture (Ethnomathematics) intersection dimension

It is important to be precise about what this synthesis does and does not establish. Table 2 and the construct map in the Results section constitute a theoretical, content-based validity argument in Messick's (1995) and Kane's (2013) sense: a logical case, built from established literature, for why these six dimensions and their second-order structure should hold together. This differs from empirical content validity evidence, which requires an independent panel of experts rating item-level content relevance against these construct definitions, for example using Lawshe's (1975) Content Validity Ratio or Polit and Beck's (2006) item-level content validity index. That expert-judgment stage has not yet been carried out; it is recommended as the immediate next step before any instrument items are drafted.

Developing indicators and an item blueprint for these six dimensions has not been undertaken at this stage of writing, since the change in construct structure calls for newly derived indicators rather than a remapping of indicators from the earlier five-dimension formulation. The intended population for the next empirical validation stage is Indonesian elementary school teachers, including teachers in Madrasah Ibtidaiyah (MI) (Noor et al., 2024; Putri, Nurwidiyanto, & Rohmah, 2020), while the sample, indicator and blueprint development, instrument piloting, and psychometric results (Standards 2 through 6, covering response process, internal structure, relations to other variables, consequences of testing, and reliability) have not yet been carried out and are recommended as the next dissertation stage.

RESULTS AND DISCUSSION

Results

K-CRMT (Knowledge of Culturally Responsive Mathematics Teaching) is defined as elementary school teachers' knowledge of how to teach mathematics, particularly the topic of fractions, in ways that are responsive to students' cultural backgrounds. Unlike an earlier formulation that treated K-CRMT as a direct sum of Mathematical Knowledge for Teaching

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(MKT) and Culturally Responsive Mathematics Teaching (CRMT), which Rubel and McCloskey (2021) argue is not enough to form a coherent construct, this article positions K-CRMT as a proposed second-order construct emerging from the intersection of three theoretical domains: Mathematics Content Knowledge, namely command of fraction content itself (Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005); Pedagogical Content Knowledge or PCK, namely general pedagogical skill for teaching content effectively (Shulman, 1986, 1987); and CRMT, namely the four dimensions of cultural responsiveness described by Aguirre and Zavala (2013). This three-circle model produces six intersection regions that serve as first-order measurable dimensions, while K-CRMT itself sits at the intersection of all three circles as a second-order factor rather than as a stand-alone parallel dimension. The literature synthesis yields these six dimensions as presented in Table 2.

Table 2 - Six First-Order Dimensions of the K-CRMT Construct

Dimension	Code	Brief Definition	Theoretical Domain	Source
Mathematics Content Knowledge	MCK	Command of fraction content itself (the five meanings of fractions, representations, operations), independent of how it is taught or its cultural context	Pure circle: content	Ball et al. (2008); Hill et al. (2005)
Pedagogical Content Knowledge	PCK	General pedagogical skill for teaching content effectively, not yet specific to mathematics or culture	Pure circle: pedagogy	Shulman (1986, 1987)
Culturally Responsive Mathematics Teaching	CRMT	Four dimensions of cultural responsiveness (identity, language, mathematical ways of thinking, social justice), not yet specific to fractions	Pure circle: culture	Aguirre & Zavala (2013)
Mathematical Knowledge for Teaching	MKT	Synthesis of content and pedagogy for fractions: knowledge of content and students, content and teaching, content and curriculum	Intersection of content ∩ pedagogy	Ball et al. (2008); Hill et al. (2005)
Ethnomathematics	EM	Recognition of fraction-related mathematical practices embedded in local culture (dividing harvest yields, weaving patterns), not yet covering how to teach them	Intersection of content ∩ culture	D'Ambrosio (1985)

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Dimension	Code	Brief Definition	Theoretical Domain	Source
Culturally Responsive Pedagogy	CRP	General pedagogical skill that is culturally responsive, not yet specific to fraction content	Intersection of pedagogy ∩ culture	Shulman (1986, 1987); Aguirre & Zavala (2013)

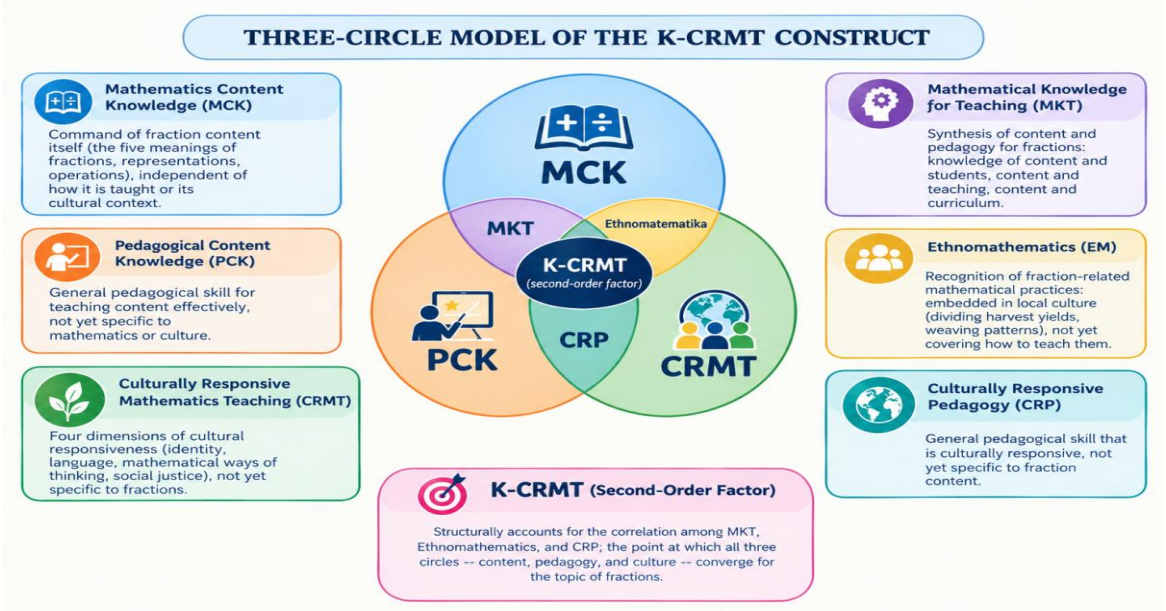

Figure 1 - Three-Circle Model of the K-CRMT Construct

The decision to model K-CRMT as a second-order factor rather than as a simple composite or formative index follows standard practice in second-order confirmatory factor modeling, where a higher-order factor is retained only when it plausibly accounts for the correlations among its first-order indicators rather than merely summarizing them (Chen, West, & Sousa, 2006; Brown, 2015). Under this logic, MKT, Ethnomathematics, and Culturally Responsive Pedagogy are treated as reflective indicators of K-CRMT: a teacher who possesses a strong underlying K-CRMT is expected to show correlated strength across all three intersection dimensions, rather than being able to substitute high performance in one dimension for low performance in another, as a formative or purely additive model would allow. This reflective second-order specification is also what distinguishes the current three-circle model from the earlier five-dimension formulation it replaces, in which the dimensions were treated as a flatter, less theoretically constrained list; whether the data ultimately support this reflective structure over a competing bifactor specification is itself an empirical question that belongs to Standard 3 (internal structure) at the next dissertation stage and is not resolved by the conceptual argument alone.

Three structural alternatives were considered before settling on this second-order reflective specification. A correlated-factor model would treat MKT, Ethnomathematics, and Culturally Responsive Pedagogy as three separate, merely correlated factors with no overarching factor; this leaves their correlation theoretically unexplained, which is precisely the gap a second-order factor is meant to close. A formative (composite) model would treat K-CRMT as caused by, rather than causing, MKT, Ethnomathematics, and Culturally Responsive Pedagogy, which implies a teacher could compensate for weakness in one dimension with strength in another; this contradicts the

substantive claim developed here, namely that the three intersection dimensions are correlated manifestations of one shared underlying capacity rather than interchangeable, substitutable components. The second-order reflective model is preferred because it is the only one of the three that matches the conceptual derivation itself: since MKT, Ethnomathematics, and Culturally Responsive Pedagogy are each built from the same three pure circles (Mathematics Content Knowledge, Pedagogical Content Knowledge, and CRMT), a shared higher-order cause is the structural signature this derivation implies (Chen, West, & Sousa, 2006; Brown, 2015), not an assumption imposed on the data after the fact.

Figure 1 visually shows the derivation process behind these six dimensions, with each definition box reproducing the same content-based definition reported in Table 1. The three pure circles are built directly from three source frameworks: command of fraction content (Mathematics Content Knowledge), the general pedagogical skill Shulman (1986, 1987) formulated as Pedagogical Content Knowledge, and the four dimensions of cultural responsiveness that Aguirre and Zavala (2013) describe as CRMT. The three pairwise intersection regions are derived by applying set logic to these three circles: the intersection of content and pedagogy yields MKT (Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005), the intersection of content and culture yields Ethnomathematics (D'Ambrosio, 1985), and the intersection of pedagogy and culture yields Culturally Responsive Pedagogy. This decompose-and-reassemble procedure, breaking a combined construct (MKT) down to its theoretical roots and then rebuilding it alongside another construct under one umbrella framework, follows the same strategy as Ogodo (2024), who integrated content knowledge, pedagogical content knowledge, and technological pedagogical content knowledge into a single umbrella construct called Culturally Responsive Pedagogical Knowledge (CRPK) for diverse STEM classrooms. The triple intersection region, where all three pure circles meet at once, is modeled as K-CRMT, a second-order factor that structurally accounts for the correlation among MKT, Ethnomathematics, and CRP, rather than as a fourth, stand-alone region.

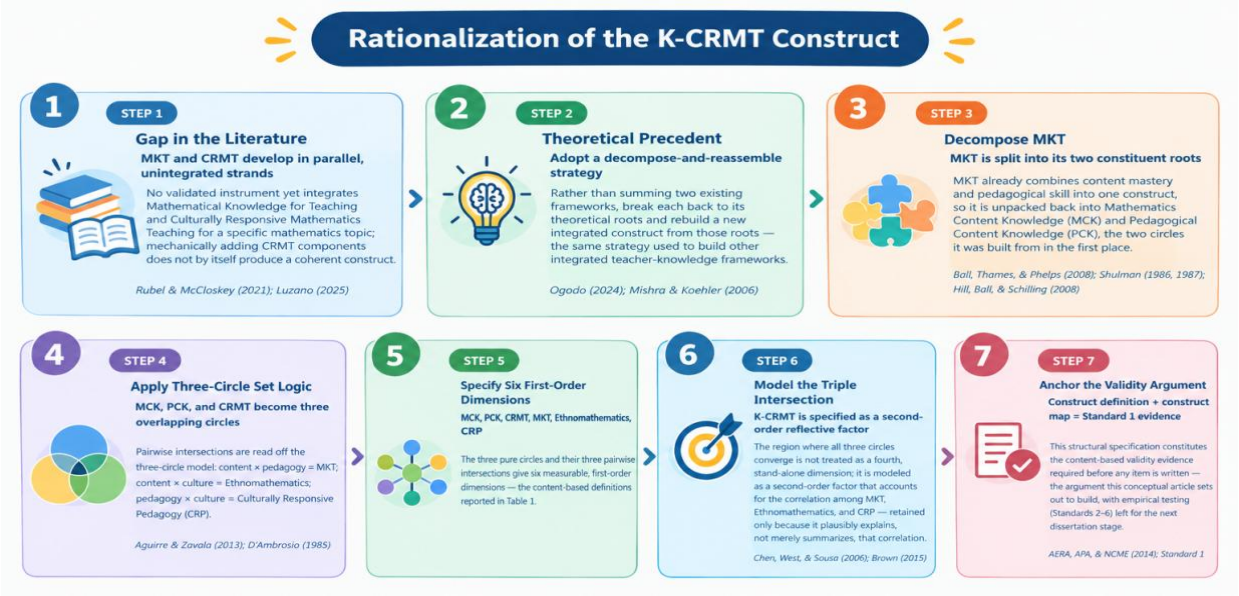

Figure 2 - Concept Map of the K-CRMT Rationalization

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Where Figure 1 shows what the K-CRMT construct consists of, Figure 2 shows why and how it was built that way, making explicit the rationalization that Figure 1 and the surrounding text argue for but do not themselves sequence as a single chain. The chain opens from the gap Rubel and McCloskey (2021) and Luzano (2025) identify, namely that no validated instrument yet integrates MKT and CRMT for a specific mathematics topic and that summing the two frameworks mechanically does not by itself yield a coherent construct. It resolves that gap by adopting the same decompose-and-reassemble strategy Ogodo (2024) and Mishra and Koehler (2006) used to build other integrated teacher-knowledge frameworks: MKT is unpacked back into the Mathematics Content Knowledge and Pedagogical Content Knowledge roots it was built from (Ball, Thames, & Phelps, 2008; Shulman, 1986, 1987; Hill, Ball, & Schilling, 2008), those two roots are joined by CRMT (Aguirre & Zavala, 2013) into a three-circle model, and three-circle set logic is applied to read off the three pairwise intersections, MKT, Ethnomathematics (D'Ambrosio, 1985), and Culturally Responsive Pedagogy, that together with the three pure circles give the six first-order dimensions in Table 1. The chain closes with the two moves that distinguish a conceptual article from a simple diagram: specifying the triple intersection as a second-order reflective factor rather than a fourth stand-alone dimension, on the grounds that a higher-order factor is retained only when it plausibly accounts for the correlation among its indicators rather than merely summarizing them (Chen, West, & Sousa, 2006; Brown, 2015), and anchoring that specification in Standard 1 of the AERA, APA, and NCME (2014) Standards as the content-based validity evidence this article sets out to provide.

Discussion

The six K-CRMT dimensions formulated in Table 1 show that integrating MKT and CRMT cannot be done by simply placing the two frameworks side by side; it requires tracing each back to its theoretical roots, as Rubel and McCloskey (2021) emphasize, so that MKT, Ethnomathematics, and Culturally Responsive Pedagogy each emerge as an intersection of two of the three pure circles rather than as a separate addition. This decomposition is not an ad hoc choice: three precedents converge on the same strategy of breaking a combined construct into its theoretical roots before reassembling it as a higher-order framework, namely Hill, Ball, and Schilling (2008), who separated mathematics teachers' content-pedagogical knowledge into interrelated components of knowledge about content and knowledge about students; Mishra and Koehler (2006), who built technological pedagogical content knowledge from the intersection of three simpler teacher-knowledge domains; and Ogodo (2024), who broke technological pedagogical content knowledge down into its constituent components before recombining them with cultural awareness into Culturally Responsive Pedagogical Knowledge.

Positioning K-CRMT against its two closest structural precedents helps sharpen what is genuinely new about it. Mishra and Koehler's (2006) Technological Pedagogical Content Knowledge model demonstrates that a three-circle, intersection-based structure can productively organize teacher knowledge, but its third circle is technology rather than culture, and its application is topic-general rather than anchored to one specific mathematics concept. Ogodo's (2024) Culturally Responsive Pedagogical Knowledge framework is structurally closer, since it also folds cultural responsiveness into a content-pedagogy base, but it operates at the level of STEM teaching broadly, without committing to a specific topic such as fractions or to an explicit three-circle, Venn-diagram-based derivation of its dimensions. K-CRMT's specific contribution,

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relative to both precedents, is narrowing the third circle to culturally responsive mathematics teaching specifically and anchoring the entire construct to one cognitively demanding mathematics topic, which is what allows the six first-order dimensions in Table 1 to be defined with fraction-specific examples, such as the five meanings of fractions and fraction-related ethnomathematical practices, rather than remaining at a level of abstraction general enough to require further topic-specific operationalization before it could be measured.

This comparison also makes explicit what K-CRMT can account for that MKT or CRMT alone cannot. MKT specifies what a teacher must know about mathematical content and pedagogy but is silent on cultural responsiveness, so a teacher could score highly on MKT while still teaching fractions in a culturally neutral or exclusionary way. CRMT specifies the cultural dimensions of responsive teaching but is silent on mathematical content and pedagogical skill, so a teacher could be highly attentive to students' cultural backgrounds while still holding a weak or incomplete grasp of fraction content itself. K-CRMT names the second-order capacity that MKT and CRMT jointly but separately leave unaccounted for: a teacher's ability to draw on mathematical content knowledge, pedagogical skill, and cultural responsiveness together when teaching fractions, rather than treating strength in one domain as a substitute for the other two. Neither MKT nor CRMT, taken alone, specifies this joint capacity, which is the gap K-CRMT is proposed to address.

The Culturally Responsive Mathematics Teaching foundation of this framework is further supported by a decade of equity-based mathematics teacher education research. Roth McDuffie, Foote, Bolson, Turner, Aguirre, Bartell, Drake, and Land (2014) show that video analysis can help prospective teachers notice their students' multiple mathematics knowledge bases, while Parker, Bartell, and Novak (2017) document that secondary mathematics teachers' conceptions of knowing their students develop gradually rather than all at once. Both findings reinforce the argument that CRMT is not merely an attitude or good pedagogical intent but a competency that can and needs to be measured explicitly, consistent with Louie's (2017) warning noted earlier.

The construct validity argument presented in this section follows the view of validity as a single, unified argument rather than a collection of separate validity types (Messick, 1995; Kane, 2013), and it specifically satisfies Standard 1 of the six AERA, APA, and NCME (2014) Standards, namely content-based validity evidence, through the construct definition and construct map for the six first-order dimensions and one second-order factor. Standards 2 through 6, namely response process, internal structure, relations to other variables, consequences of testing, and reliability/precision, have not yet been carried out and stand as the agenda for the next dissertation stage, including developing indicators and an instrument blueprint for the six newly formulated dimensions, readability testing and cognitive interviews, hierarchical confirmatory factor analysis of this two-order structure, and instrument reliability estimation.

Beyond satisfying Standard 1 through construct definition and a construct map, the second-order reflective structure argued for in the Results section carries a substantive theoretical claim: MKT, Ethnomathematics, and Culturally Responsive Pedagogy are not three independent competencies a teacher could develop in isolation, but three correlated manifestations of one underlying integrated capacity (Chen, West, & Sousa, 2006). This claim is falsifiable once empirical data are collected, since a formative or bifactor model that fits the data better than the

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second-order model would undercut it. Flagging this as a testable rather than assumed property of the model is itself part of the content-based validity evidence this article aims to provide, since content-based evidence under the AERA, APA, and NCME (2014) Standards concerns whether the construct is well specified prior to data collection, not only whether its items look relevant on their face.

The decision to anchor K-CRMT to fractions rather than leaving it topic-general also sets a boundary condition on the construct's claims. Fractions are used here precisely because they are unusually demanding on all three circles at once: they are cognitively difficult for students to grasp as a mathematical object (Kieren, 1976; Behr, Lesh, Post, & Silver, 1983), their difficulty persists across age groups and school systems rather than resolving with more instruction alone (Siegler, Fazio, Bailey, & Zhou, 2013), and they remain one of the most persistent obstacles in school mathematics internationally (Lortie-Forgues, Tian, & Siegler, 2015), which together make fractions a demanding proving ground for whether an integrated content-pedagogy-culture construct can be specified coherently at all. This also means the six-dimension structure and its specific definitions in Table 1 should not be assumed to transfer unchanged to other mathematics topics, such as geometry or early number sense, without separate theoretical and empirical work; what should transfer is the three-circle decomposition logic itself, which Ogodo's (2024) and Mishra and Koehler's (2006) parallel constructs suggest is a more general pattern in teacher-knowledge theorizing, even though the content of each circle, and therefore of each dimension, remains topic- and context-specific by design.

The contribution of this K-CRMT framework answers Bartell, Drake, Roth-McDuffie, Aguirre, Turner, and Foote’s (2019) call for validated frameworks and instruments that measure teacher knowledge in an integrated way. This proposed integrated construct’s novelty claim rests on the scope of Luzano’s (2025) scoping review, which surveyed the literature on culturally responsive mathematics teaching strategies broadly rather than focusing specifically on measurement instruments or on decomposing MKT back to Shulman’s roots. Given that scope, the possibility of a similar framework existing outside the review cannot be fully ruled out, and this novelty claim needs further verification through a more thorough literature review at the next dissertation stage. A second limitation concerns the Theory Synthesis method itself: because this approach depends on the researcher’s judgment in selecting which frameworks to trace back to shared roots and how to recombine them, the resulting construct map is open to alternative decompositions that a different researcher might reasonably propose (Jaakkola, 2020), and this subjectivity is not resolved by the argument presented here. A third limitation, discussed above, is that the six-dimension structure is anchored specifically to fraction teaching and should not be assumed to transfer unchanged to other mathematics topics without separate theoretical and empirical work. The main limitation of this article is that it is purely conceptual and a structural revision of an earlier formulation: no expert judgment, operational indicators, instrument piloting, or psychometric data yet exist to support the six K-CRMT dimensions empirically.

CONCLUSION

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This article makes three contributions. First, it proposes K-CRMT, a second-order construct synthesizing Mathematics Content Knowledge (Ball et al., 2008; Hill et al., 2005), Pedagogical Content Knowledge (Shulman, 1986, 1987), and Culturally Responsive Mathematics Teaching (Aguirre & Zavala, 2013) around the topic of fractions, yielding six first-order measurable dimensions, namely Mathematics Content Knowledge, Pedagogical Content Knowledge, Culturally Responsive Mathematics Teaching, Mathematical Knowledge for Teaching, Ethnomathematics, and Culturally Responsive Pedagogy, with K-CRMT as the second-order factor emerging from the intersection of all three circles. Second, this proposed construct carries theoretical significance for culturally responsive elementary mathematics teaching in Indonesia, including in Madrasah Ibtidaiyah, by naming what an integrated content-pedagogy-culture competence for fraction teaching would need to include, something neither MKT nor CRMT names on its own. Third, because the argument presented here satisfies only Standard 1 (content-based evidence) of the six AERA, APA, and NCME (2014) Standards, this proposed construct requires empirical validation, expert judgment, instrument piloting, and psychometric testing before it can be used in assessment or teacher development; it is not yet a validated instrument.

Further research is recommended to first develop indicators and an item blueprint for the six newly formulated dimensions, given that this change in construct structure replaces the earlier five-dimension formulation and has no item-level operationalization yet. After that, the next dissertation stage needs to carry out Standards 2 through 6, covering expert judgment and content validity index computation (Lawshe, 1975; or alternatively Polit & Beck, 2006), cognitive interviews with prospective respondents, including teachers from Madrasah Ibtidaiyah, exploratory and hierarchical confirmatory factor analysis of the instrument's two-order structure following second-order factor modeling procedures (Brown, 2015; Chen, West, & Sousa, 2006), tests of relations with other variables such as teaching experience and CRMT training, differential item functioning analysis to guard against bias, and instrument reliability estimation.

Potential Practical Implications

Beyond its contribution to validity theory, this proposed framework carries several potential practical implications, conditional on empirical and psychometric validation at the next dissertation stage rather than actionable immediately. For pre-service teacher education, the six-dimension structure suggests that content knowledge, pedagogical content knowledge, and cultural responsiveness for a specific mathematics topic such as fractions should be taught as an integrated unit rather than across separate, unconnected courses, directly answering Bartell, Drake, Roth-McDuffie, Aguirre, Turner, and Foote's (2019) call for teacher education that measures and develops these domains together. For in-service professional development, a validated instrument built on this construct map could support modular training that diagnoses which of the six dimensions, rather than which of the three source frameworks, a teacher most needs to strengthen, which is a finer-grained diagnostic than MKT or CRMT alone can offer on its own. For instructional materials, the Ethnomathematics and Culturally Responsive Pedagogy dimensions point toward a concrete design brief: fraction-teaching materials that draw on local cultural practices Indonesian elementary teachers already have access to, following the funds-of-knowledge tradition (Moll, Amanti, Neff, & Gonzalez, 1992; Civil, 2007) and ethnomathematics

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scholarship (D'Ambrosio, 1985; Rosa & Orey, 2013) already reviewed in this article. For school-level supervision, once indicators exist, the six dimensions could inform formative classroom-observation rubrics that give mathematics coordinators and school leaders a shared vocabulary for feedback that is more specific than general pedagogical competence. None of these applications are warranted by the present conceptual argument alone; they depend on the indicator development, expert judgment, and psychometric validation recommended above, and are presented here as the direction this line of work points toward rather than as conclusions this article itself establishes.

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